upper Parts, under which the Colours have appear'd. I have taken notice of this ſo very often, that I can hardly look upon it to be accidental, and if it ſhould prove true in general, it will bring the diſquiſition into a narrow compaſs; for it will ſhew that this Effect depends upon ſome Property, which the Drops retain, whilſt they are in the upper part of the Air, but loſe as they come lower, and are more mix'd with one another.

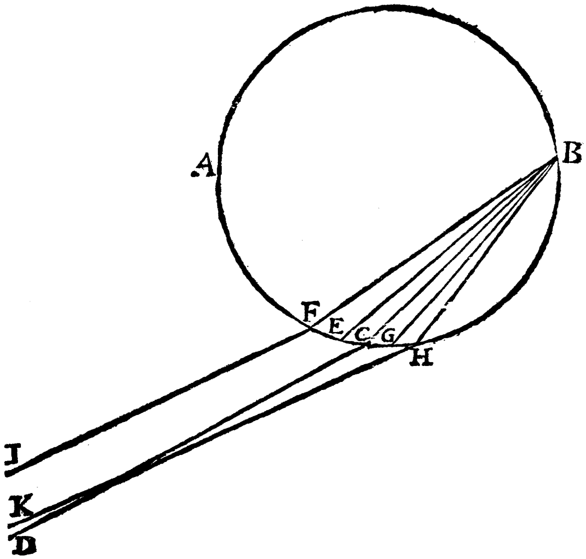
Petworth, Oct. 13.
1722.

V. *A Letter to Dr.* Jurin, *Coll. Med.* Lond. *Soc. & Secr. R. S. concerning the abovementioned Appearance in the Rainbow, with ſome other Reflections on the ſame Subject. By* Henry Pemberton, *M. D. R. S. S.*

SIR,

UPON your communicating to me the curious Obſervations, your Friend Dr. *Langwith* had made on the Rainbow, I inform'd you thoſe Appearances might, I thought, be explain'd by the Diſcoveries, the Great Sir *Iſaac Newton* had made in the Subject of Light and Colours, in his wonderful Treatiſe of *Optics*. As you ſeemed not diſpleaſed with what I mentioned to you in relation to this Matter by word of mouth, you deſired that I would ſet down in writing my Thoughts thereupon, which I have here accordingly done in the following manner.

R Let



Let AB repreſent a Drop of Rain, B the Point from whence the Rays of any determinate Species being reflected to C, and afterwards emerging in the Line CD, do proceed to the Eye, and cauſe the Appearance of that Colour in the Rainbow, which appertains to this Species. It is obſerved by Sir *Iſaac Newton* [a], that in the Reflection of Light, beſides what is reflected regularly, ſome ſmall part of it is irregularly ſcattered every way. So that from the Point B, beſides the Rays that are regularly reflected from B to C, ſome ſcattered Rays will return in other Lines, as in B E, B F, B G, B H, on each Side the Line B C. Further it muſt be noted from Sir *Iſaac Newton* [b], that the Rays of Light in their Paſſage from one Superficies of a refracting *Medium* to the other undergo alternate

[a] Optics, *Book* II. *Part* 4. [b] Ibid. *Part* III. *Prop.* xij.

Fits

Fits of eaſy Tranſmiſſion and Reflection, ſucceeding each other at equal Intervals; inſomuch that if they reach the further Superficies in one ſort of thoſe Fits, they ſhall be tranſmitted; if in the other kind of them, they ſhall rather be reflected back. Whence the Rays that proceed from B to C, and emerge in the Line CD, being in a Fit of eaſy Tranſmiſſion, the ſcattered Rays that fall at a ſmall Diſtance without theſe on either ſide, (ſuppoſe the Rays, that paſs in the Lines BE, BG) ſhall fall on the Surface in a Fit of eaſy Reflection, and ſhall not emerge; but the ſcattered Rays, that paſs at ſome Diſtance without theſe laſt, ſhall arrive at the Surface of the Drop in a fit of eaſy Tranſmiſſion, and break through that Surface Suppoſe theſe Rays to paſs in the Lines BF, BH; the former of which Rays ſhall have had one Fit more of eaſy Tranſmiſſion, and the latter one Fit leſs, than the Rays that paſs from B to C. Now both theſe Rays, when they go out of the Drop, will proceed by the Refraction of the Water in the Lines FI, HK, that will be inclined almoſt equally to the Rays incident on the Drop, that come from the Sun, but the Angles of their Inclination will be leſs than the Angle, in which the Rays emerging in the Line CD are inclined to thoſe incident Rays. And after the ſame manner Rays ſcattered from the Point B, at a certain Diſtance without theſe, will emerge out of the Drop, while the intermediate Rays are intercepted; and theſe emergent Rays will be inclined to the Rays incident on the Drop in Angles ſtill leſs than the Angles, in which the Rays FI and HK are inclined to them; and without theſe Rays will emerge other Rays, that ſhall be inclined to the incident Rays in Angles yet leſs. Now by this means will be formed of every kind of Rays, beſides the principal Arch which goes to the Formation of the Rainbow, other Arches, within every one of the principal, of the ſame Colour, though much more faint: and this for divers Succeſſions, as long as theſe weak Lights, which in every Arch grow more and more obſcure, ſhall continue viſible. Now as the Arches produced by each Colour will be variouſly mixed together, the diverſity of Colours obſerved by Dr. *Langwith* may well ariſe from them.

The preciſe Diſtances between the principal Arch of each reſpective Colour and theſe fainter correſpondent Arches depend on the Magnitude of the Drops of Rain. In particular, the ſmalleſt Drops will make the ſecondary Arches of each Species at the greateſt Diſtance from their reſpective principal, and from each other. Whence, as the Drops of Rain increaſe in falling, theſe Arches near the Horizon by their great Nearneſs to their reſpective principal Arches become inviſible.

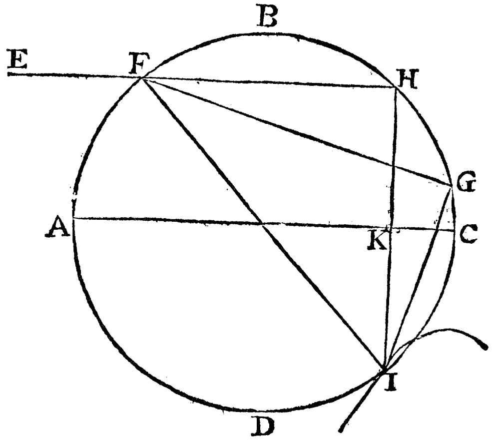
AND now, Sir, we are upon the Rainbow, I ſhall here take the Freedom of ſetting down two Propoſitions, which I have formerly conſidered, relating to this Subject. For the greater Brevity I ſhall deliver them under the Form of Poriſms; as, in my Opinion, the Ancients called all Propoſitions treated by Analyſis only.

PROPOSITION I.

In a given refracting Circle, whoſe refracting Power is given, the Ray is given in Poſition, which paſſing parallel to a given Diameter of the Circle is refracted by that Circle to a Point given in the Circumference of it.

Let

Let A B C D be the given Circle, the given Diameter A C, and given Point G; and let the Ray E F, parallel to A C, be refracted to G. I ſay E F is given in Poſition.



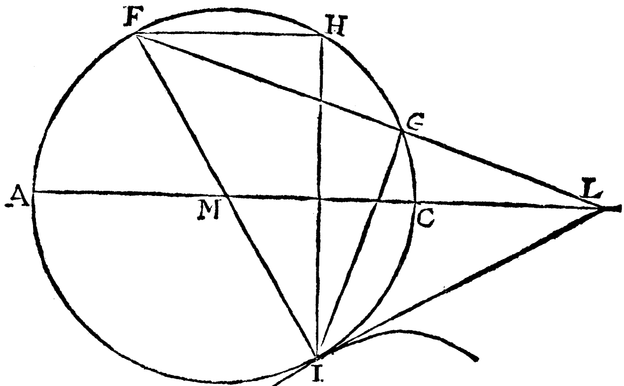
Produce E F to H, and draw the Diameter F I, drawing likewiſe I K H, I G. Then is H F I the Angle of Incidence, and G F I the refracted Angle; ſo that I H being perpendicular to F H and I G perpendicular to F G, I H is to I G as the Sine of the Angle of Incidence to the Sine of the refracted Angle, and the *Ratio* of I H to I G is given, as likewiſe the *Ratio* of I K to I G. Therefore I K being perpendicular to A C the Point I is in a Conic Section given in Poſition, whoſe Axis is perpendicular to A C, and one of its *Foci* is the Point G [a]. Conſequently the Points I and F are given, and laſtly the Ray E F given in Poſition.

[a] *See* Papp. l. 7. prop. 238. Milnes Conic. part. 4. prop. 9.

DETER-

DETERMINATION.

It is evident, that this conic Section, may either cut the Circle in two Points, touch it in one Point, or fall wholly without it. Therefore let the Section touch the Circle in the Point I, and let I L touch both the Section and the Circle in the same point I. Then G L being joined, the Angle under I G L on ac-



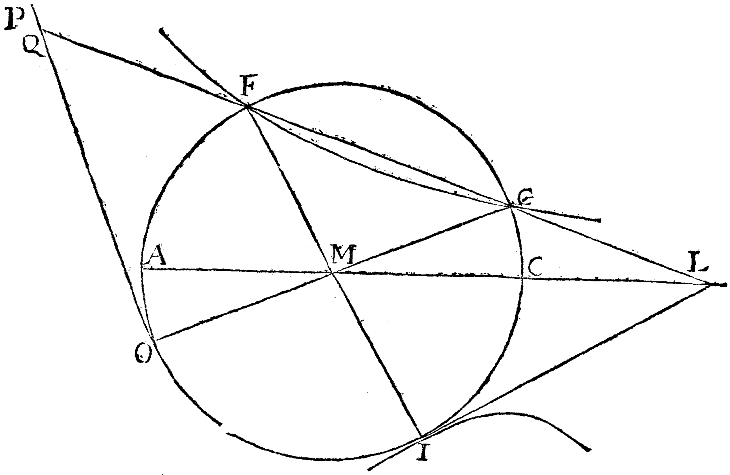
count of the conic Section is a right one [a], so that F G L is one continued right Line, and I F is to I L as F G to G I; as likewise, M being the Center of the Circle, M I to I L, or F H to H I, as F G to twice G I, because M I is to I F as G I to twice G I. Hence by Permutation F H is to F G as H I to twice G I; that is, as the Sine of the Angle of Incidence to twice the Sine of the refracted Angle.

[a] De la Hire Conic. lib. 8. prop. 23.

Moreover F H being to H I as F G to twice G I, the Square of F H will be to the Square of H I, as the Square of F G to four times the Square of G I. Therefore, by Composition, as the Square of F H to the Square of F I or of A C, so is the Square of F G to the Square of F I together with three times the Square of G I, and so likewise is the Excess of the Square of F G above the Square of F H, which equals the Excess of the Square of I H above the Square of I G, to three times the Square of G I; for as one Antecedent to one Consequent, so is the difference of the Antecedents to the difference of the Consequents. Hence in the last place, the Square of half F H will be to the Square of A M, as the Excess of the Square of I H above the Square of I G to three times the Square of I G, or as the Excess of the Square of the Sine of Incidence above the Square of the Sine of Refraction, to three times the Square of the Sine of Refraction.

Another DETERMINATION.

Draw the Diameter G O and the Tangent O P, meeting G F produced in Q: then the Angle under I F G is equal to the Angle under O G F, the Angle



under F I L equal to that under G O Q, both being right, and F I is equal to G O; whence the Triangles G O Q, F I L are ſimilar and equal; ſo that G Q is equal to F L, and the Point F in an *Hyperbola* paſſing through G, whoſe Aſymptotes are A C and O P [a].

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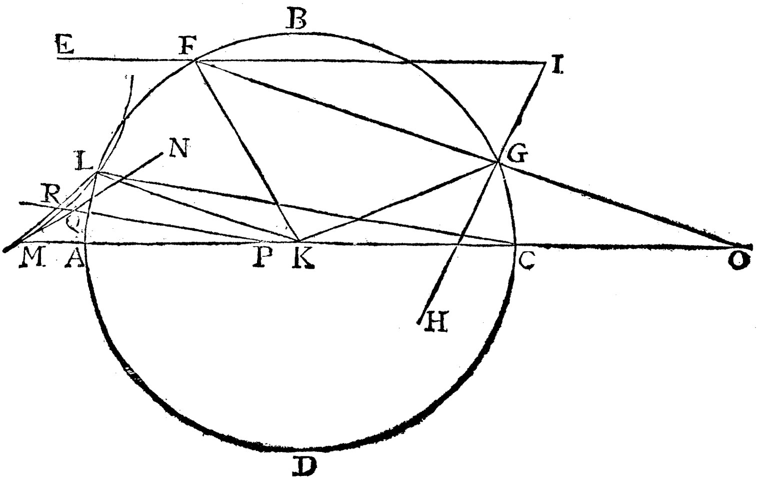
[a] Apoll. Conic. l. 2. prop. 8.

PROPO-

PROPOSITION II.

A refracting Circle and its refracting Power being given, the Ray is given in Position, which, passing parallel to a given Diameter of the Circle, after its Refraction, is so reflected from the farther Surface of the Circle, as to be inclined to its incident Course in a given Angle.

Let A B C D be the given Circle; let A C be the given Diameter, E F the incident Ray parallel to it, which being refracted into the Line F G shall so be reflected from the Point G in the Line G H, that E F and H G being produced, till they meet in I, the Angle under E I H shall be given.



Let K be the Center of the Circle, and K F, K G be joined; let the Semidiameter L K be parallel to the

 refracted

refracted Ray F G, and M K being taken to the Semidiameter of the Circle in the *Ratio* of the Sine of Incidence to the Sine of Refraction; let L M be joined, and lastly make the Angle under K M N equal to half the given Angle under E I H. This being done, if F G be produced to O, F O shall be to K O as the Sine of the Angle of Incidence to the Sine of the refracted Angle, that is as M K to K L; in so much that K L being parallel to F O, and the Angle under M K L equal to that under F O K, the Angle under M L K shall be equal to that under F K O, and the Angle under K M L equal to that under K F O equal to that under F G K or half that under F G H, whence the Angle under K M N being equal to half the Angle under F I H, the residuary Angle under N M L will be equal to half the Angle under I F G or to half that under M K L. Therefore L C being drawn, the Angle under L M N will be equal to that under M C L; and in the last place, if M C be divided into two equal Parts in P, and P Q R be drawn parallel to C L, the Angle under Q M R will be equal to that under R P M, and the Triangles Q M R, M P R similar, so that the Rectangle under P R Q shall be equal to the Square of M R. Whence R L being equal to M R, the Point L shall be in an equilateral *Hyperbola*, touching the Line M N in the Point M, and having the Point P for its Center [a]. But this *Hyperbola* is given in Position, and consequently the Point L, the Angle under M L K, and the equal Angle under C K F will be given, and therefore the Ray E F is given in Position.

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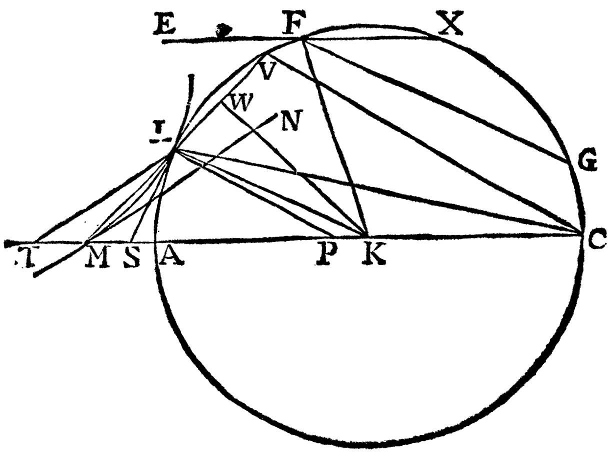
[a] Apoll. Conic. lib. 1. prop. 37. *compared with* lib. 7. prop. 23.

D E-

DETERMINATION.

Let the *Hyperbola* touch the Circle in the Point L, and let their common Tangent be L S; draw L T parallel to M N, ſo as to be ordinately applied in the *Hyperbola* to the Diameter C M. Whence L S touching the *Hyperbola* in L, P T will be to T L as T L to T S [a], and the Angle under T S L equal to that under T L P, but as the Angle under S C L is equal to that under N M L, the ſame is equal to the Angle under T L M; therefore the Angle under S L C is equal to the Angle under M L P. Farther, M L being produced to V and V C joined, the Angle under L V C is



equal to that under S L C, by reaſon that L S touches the Circle in L; hence the Angles under L V C and under M L P are equal, L P, V C are parallel, and

[a] Apoll. Conic. lib. 1. prop. 37. *compared with* lib. 7. prop. 23.

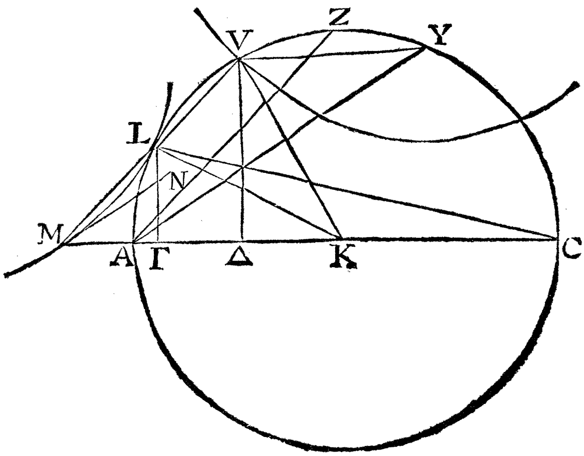
M P being equal to P C, M L is equal to L V ; and K W being let fall perpendicular to LV, M W is equal to three times L W. But now if the incident Ray E F be produced to X, the Angle under M L K being equal to that under C K F, or to that under E F K, F X ſhall be equal to LV, equal to twice L W ; and the Angle under K M L being equal to that under K F G ; ſince K W is perpendicular to M W, F G ſhall be to twice M W as M K to K F, or as the Sine of Incidence to the Sine of Refraction : whence M W being equal to three times L W, F X ſhall be to F G as the Sine of Incidence to three times the Sine of Refraction.

Moreover, M W being equal to three times L W, the Square of M W will be equal to nine times the Square of L W, and the Rectangle under V M L, or the Rectangle under C M A, that is, the Exceſs of the Square of K M above the Square of K A, will be equal to eight times the Square of LW ; therefore the Square of L W or the Square of half F X will be to the Square of K L, or of KA, as the Exceſs of the Square of K M above the Square of K A to eight times the Square of K A, that is, as the Exceſs of the Square of the Sine of Incidence, above the Sine of Refraction to eight times the Square of the Sine of Refraction.

Another

Another DETERMINATION.

Draw A Y parallel to M N, and A Z parallel to M V: then is the Angle under Y A *Z*, equal to that under L M N, which is equal to that under L C A; whence the Arches A L, Y *Z* are equal; but the



Arches A L, V Z are likewise equal, because LV, AZ are parallel, therefore Y V being joined, and L Γ drawn perpendicular to A C, the Chord V Y shall be the double of L Γ; but V Δ being likewise let fall perpendicular to A C, because M V is the double of M L, V Δ shall be the double of L Γ; and therefore V Δ and V Y shall be equal; whence the point V shall be in a *Parabola*, whose *Focus* is the Point Y, its Axis perpendicular to A C, and the *Latus rectum*, belonging to that Axis, equal to twice the perpendicular let fall from Y upon A C [a]. But if K V be joined, the

[a] Vide de la Hire Sect. Conic. lib. 8. prop. 13.

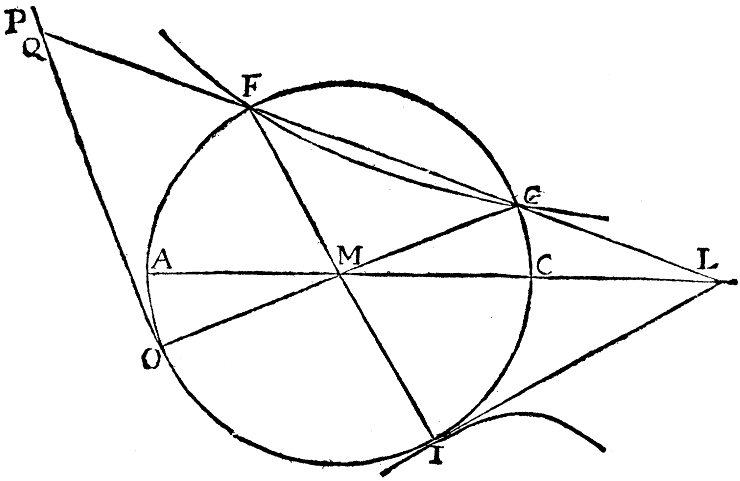
Angle

Angle under L K V is equal to twice the Complement to a right Angle of the Angle under K L V, which is equal to the Angle of Incidence, and exceeds the refracted Angle by the Angle under A K L.

THE Determinations of these two Propositions, have relation to the first and second Rainbow; those of the first Proposition respecting the interior, and those of the second the exterior. The first Determinations of these two Propositions assign the Angles, under which each Rainbow will appear in any given refracting Power of the transparent Substance, by which they are produced; the latter Determinations of these Propositions teach how to find the refracting Power of the Substance, from the Angles under which the Rainbows appear; the Angle under C M G, in the Determinations of the first Proposition, being half the Angle which measures the Distance of the interior Bow from the Point opposite to the Sun; and in the Determinations of the second Proposition, the Angle under C M N is half the Complement to a right Angle of half the Angle that measures the Distance of the exterior Bow, from the Point opposite to the Sun. But whereas these latter Determinations require solid Geometry, it may not be amiss here to shew how they may be reduced to Calculation, seeing the Observation of these Angles, as the learned Dr. *Halley* has already remark'd [a], affords no inconvenient Method of finding the refracting Power of any Fluid, or indeed of any transparent Substance, if it be formed into a spherical

[a] *Philosoph. Transact.* No. 267. *pag.* 722.

or cylindrical Figure. For this purpose therefore I

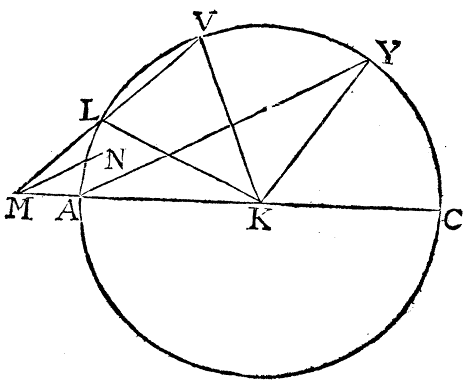


have found, that in the latter Determination of the first Proposition, if the Sine of the Angle under CMG be denoted by *a*, the Tangent of the Complement of this Angle to a right one be denoted by *b*, and the Secant of this Complement by *c*; the Root of this Equation $z^3 - 3aaz = 2aa \times \overline{2c - a}$ will exceed the Sine of the Angle under F M A, that is the Sine of the Angle of Incidence, by the Sine of the Angle under C M G; and the Sine of the Angle under FMO, which is double the refracted Angle, will be the Root of this Equation $x^3 + 3aax = 4aab$; this Angle being acute, when the Tangent of the Angle under C M G is less than half the *Radius*, or when the Angle itself is less than 26 *degr.* 33′. 54″. 11‴, and when this Tangent is more than half the *Radius*, the Angle under O M F is obtuse.

The

The Roots of theſe cubic Equations are found by ſeeking the firſt of two mean Proportionals, between each of the verſed Sines appertaining to the Arches C G, A G, and the Sine of thoſe Arches, counting from the verſed Sines; for the Sum of theſe two mean Proportionals is the Root of the former Equation, and the difference between them the Root of the latter; as may be collected from *Cardan*'s Rules.

And hence likewiſe if the firſt and laſt of the five mean Proportionals, between the Sine and Coſine of half the Angle under C M G be found, twice the Sum of the Squares of theſe mean Proportionals applied to the *Radius* exceeds the Sine of the Angle of Incidence by the Sine of the Angle under C M G; and twice the difference of the Squares of the ſame mean Proportionals applied to the *Radius* is equal to the Sine of double the refracted Angle. Moreover this double of the refracted Angle exceeds the Angle of Incidence by the Angle under C M G.



In the latter Determination of the ſecond Propoſition draw K Y, and A Y being parallel to M N, the Angle under C K Y will be equal to twice the Angle under

under CMN, that is equal to the Complement of half the Diſtance of the exterior Rainbow from the Point oppoſite to the Sun. Then putting *a* for the *Radius* AK, and *b* for the Sine of the Angle under CKY, the Sine of the Angle under AKV will be the Root of this Equation $y^4 + 4by^3 - 8aaby + 4aabb = 0$. But the Angle of Incidence and Refraction may alſo be found as follows.

Let two mean Proportionals between the *Radius* and the Sine of the Angle under CKY be found, then take the Angle, whoſe Coſine is the firſt of theſe mean Proportionals, counting from the *Radius*; and alſo the Angle, whoſe Sine together with the ſecond mean Proportional ſhall be to the *Radius* as the Coſine of the Angle under CKY to the Sine of the Angle before found. The Sum of theſe three Angles is double the Complement to a right one of the Angle under AKL, the Angle under KML, or the refracted Angle, being equal to half the Sum of this Angle under AKL and the Angle under CKY; as in the laſt Place the Angle under KLV, that is the Angle of Incidence, equal to the Sum of the Angles under KML and under MKL.

I need not obſerve, that the geometrical Methods of deducing theſe Angles of Incidence and Refraction from the Angle meaſuring the Diſtance of each Rainbow from the Point oppoſite to the Sun, afford very expeditious mechanical Conſtructions.